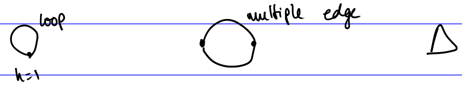
HW2

5. Lots of you had trouble with the induction.



etc.

Note that the $k=2$ case is false! So multiple edges are not allowed. Loops are not allowed in W either since they're cycles.

Closed walks of length 1 can be ignored since they're loops. So assume the walk has at at least two steps.

Direct: $W = v_1 \cdots v_k$, $v_1 = v_k$ (closed)

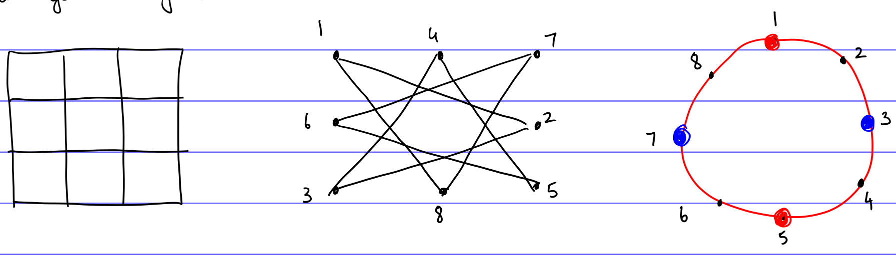
Find $i<j$ st $v_i = v_j$

the # of vertices between v_i and v_j on W is

From your Textbook defn 1.1.15: A cycle is a graph with $|V(G)| = |E(G)|$, and whose vertices can be placed around a circle, whose vertices are adjacent iff they appear consecutively on a circle.

No vertices are repeated between v_i and v_j because by definition this is the minimal closed walk. If there is more than one vertex between v_i and v_j, then this is a cycle of length at least 3. So there must be exactly one vertex z between v_i and v_j. This edge $v_i z, z v_j$ must be repeated twice.

9. Most of you got this right.



This is a proof by picture. The graph of moves is an 8 cycle. It's easy to see that whenever two knights of the same color are in adjacent corners, the other knight must be sandwiched between them in one of the sites $\{2,4,6,8\}$ which is not a corner.